

### BTDT Exam MFE Sample 2

For a 6-month European gap call option on a stock of Szczydzycze, a Polish financial conglomerate traded on the Warsaw Stock Exchange:

- (i) The stock's price is 65.
  - (ii) The trigger is 70.
  - (iii)  $\sigma = 0.25$ .
  - (iv) The continuous annual dividend rate for the stock is 0.02.
  - (v) The risk-free rate is 5% annual, continuously compounded.
  - (vi) Options are priced using the Black-Scholes formula.
- Determine the strike price to make the option price equal to 0.

- A. 78.53      B. 79.05      C. 79.77      D. 80.02      E. 81.25

Solution.

The price of a gap call is a modification of the Black-Scholes formula:

$$C = C(S, K_1, K_2, \sigma, r, T, \delta) = Se^{-\delta T} N(d_1) - K_1 e^{-rT} N(d_2),$$

where  $d_1 = \frac{\ln\left(\frac{Se^{-\delta T}}{K_2 e^{-rT}}\right) + \frac{1}{2}\sigma^2 T}{\sigma\sqrt{T}}$ ,  $d_2 = d_1 - \sigma\sqrt{T}$ . Here,  $K_2 = 70$  and  $K_1$  needs to be

found, given that  $C = 0$ . We have the following parameters of the Black-Scholes formula

$$d_1 = \frac{\ln\left(\frac{65e^{-0.02 \cdot \frac{1}{2}}}{70e^{-0.05 \cdot \frac{1}{2}}}\right) + \frac{1}{2} \cdot 0.25^2 \cdot 0.5}{0.25\sqrt{0.5}} \approx -0.2460 \approx -0.25,$$

$$N(d_1) \approx 1 - N(-0.25) = 1 - 0.5987 = 0.4013,$$

$$d_2 \approx -0.25 - 0.25\sqrt{0.5} \approx -0.4228 \approx -0.42,$$

$$N(d_2) \approx N(-0.42) = 1 - N(0.42) = 1 - 0.6628 = 0.3372.$$

We set  $C = 0$  and obtain the following equation for  $K_1$ :

$$0.4013 \cdot 65e^{-0.01} - 0.3372 \cdot K_1 \cdot e^{-0.025} = 0.$$

This results in

$$K_1 = \frac{0.4013 \cdot 65e^{-0.01}}{0.3372 \cdot e^{-0.025}} \approx 78.5253.$$

Answer A.